[2] P. H. Masterman and P. J. B. Clarricoats, "Computer field-matching solution of waveguide transverse discontinuities," in *Proc. Inst. Elec. Eng.*, vol. 118, no. 1, pp. 51–63, Jan. 1971.
[3] R. Safavi-Naini and R. H. MacPhie, "On solving waveguide junction scattering problems by the conservation of complex power technique," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 337–343, Apr. 1981.
[4] R. Mittra and S. W. Lee, *Analytical Techniques in the Theory of Guided Waves*. New York: Macmillan, 1971.
[5] A. S. Omar and K. Schunemann, "Transmission matrix representation of finline discontinuities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 765–770, Sept. 1985.
[6] R. R. Mansour and R. H. MacPhie, "An improved transmission matrix formulation of cascaded discontinuities and its application to E-plane circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1490–1498, Dec. 1986.
[7] J. D. Wade and R. H. MacPhie, "Scattering at circular-to-rectangular waveguide junctions," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 1085–1091, Nov. 1986.
[8] P. Couffignal, H. Baudrand, and B. Theron, "A new rigorous method for the determination of iris dimensions in dual-mode cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1314–1320, July 1994.
[9] U. Papziner and F. Arndt, "Field theoretical computer-aided design of rectangular and circular iris coupled rectangular or circular cavity filters," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 462–471, Mar. 1993.

Comment on "Are Nonreciprocal Bi-Isotropic Media Forbidden Indeed?"

Akhlesh Lakhtakia and Werner S. Weiglhofer

***Abstract*—In a recent paper,[1] Sihvola has cast doubt on our claim that all bi-isotropic media must be reciprocal. We show that Sihvola's doubt has no rational basis.**

Consider the linear homogeneous bi-anisotropic medium whose constitutive $\tilde{\underline{\underline{\alpha}}}(\tau)$ relations are given as

$$\underline{\tilde{D}}_{,}(\underline{r},t) = \varepsilon_0 \underline{\tilde{E}}(\underline{r},t) + \int_{-\infty}^{\infty} \left[\varepsilon_0 \tilde{\underline{\underline{\chi}}}_c(\tau) \bullet \underline{\tilde{E}}(\underline{r}, t-\tau) + \tilde{\underline{\underline{\alpha}}}(\tau) \bullet \underline{\tilde{B}}(\underline{r}, t-\tau)\right] d\tau \tag{1a}$$

$$\underline{\tilde{H}}_{,}(\underline{r},t) = \varepsilon_0 \underline{\tilde{B}}(\underline{r},t) + \int_{-\infty}^{\infty} \left[\tilde{\underline{\underline{\beta}}}(\tau) \bullet \underline{\tilde{E}}(\underline{r}, t-\tau) - \frac{1}{\mu_0} \tilde{\underline{\underline{\chi}}}_m(\tau) \bullet \underline{\tilde{B}}(\underline{r}, t-\tau)\right] d\tau \tag{1b}$$

where $\tilde{\underline{\underline{\chi}}}_c(t)$ and $\tilde{\underline{\underline{\chi}}}_m(t)$ are the dyadic susceptibility kernels, while $\tilde{\underline{\underline{\alpha}}}(t)$ and $\tilde{\underline{\underline{\beta}}}(t)$are the magnetoelectric kernels. Covariance in conjunction with the mathematical consistency of the Maxwell postulates leads to the condition [1], [2]

$$Trace\left[\tilde{\underline{\underline{\alpha}}}(t) - \tilde{\underline{\underline{\beta}}}(t)\right] = 0. \tag{2}$$

A very important consequence of (2) is that all bi-isotropic media must be reciprocal [2].

Doubt has been cast by Dr. Ari H. Sihvola [3] on this conclusion, as well as on the validity of (2). We will show that his doubt does not have a rational basis. Following the publication of [2] (on which Sihvola solely bases his critical remarks), further evidence on the validity of (2) had been presented by us in several publications [4]–[8], which had been made available to Sihvola well before he finalized [3].

I. Mathematical Basis For (2)

Consider a discrete electric charge moving with a constant velocity under the influence of an electromagnetic field. This charge experiences the usual Coulomb force in a co-moving frame of reference. In the laboratory frame ($\underline{r}$,t), that force is nothing but the usual Lorentz force, which fact can be established from the Lorentz covariance of the Maxwell postulates. As a result, the primitive fields are $\underline{\tilde{E}}(\underline{r},t)$ and $\underline{\tilde{B}}(\underline{r},t)$; therefore, the induction fields must be $\underline{\tilde{D}}(\underline{r},t)$ and $\underline{\tilde{H}}(\underline{r},t)$. Consequently, these induction fields have to be expressed as functionals of the primitive fields in a material medium. For a linear homogeneous medium, (1a) and (1b) thus follow as the time-dependent constitutive relations in modern electromagnetic theory.

In a sourceless region, the Faraday equation

$$\nabla \times \underline{\tilde{E}}(\underline{r},t) = -\frac{\partial \underline{\tilde{B}}(\underline{r},t)}{\partial t} \tag{3}$$

contains only the primitive fields and is therefore not affected by the constitutive relations. The Ampere-Maxwell equation

$$\nabla \times \underline{\tilde{H}}(\underline{r},t) = -\frac{\partial \underline{\tilde{D}}(\underline{r},t)}{\partial t} \tag{4}$$

contains the induction fields. When (1a) and (1b) are substituted into (4) and the result is compared with (3), a redundancy emerges. This redundancy is removed by (2), which has covariance [1] as well as uniqueness [4] proofs.

Equation (2) has also been extended to nonhomogeneous media [9].

Sihvola has not been able to prove that (2) is mathematically incorrect; indeed, he has not even suggested that is so. Instead, he has chosen to present two media that can possibly refute (2). Let us now show that his physical counter-arguments are unsustainable.

II. Tellegen Medium

Many decades ago, Tellegen conceived manufacturing a nonreciprocal bi-isotropic medium by randomly dispersing inclusions of a special type in an appropriate host medium. Each inclusion would be made of a glued pair of two parallel dipoles, one magnetic and the other electric. The resulting medium would be isotropic: $\tilde{\underline{\underline{\chi}}}_0(t) = \tilde{\chi}_c(t)\underline{\underline{I}}$, $\tilde{\underline{\underline{\chi}}}_m(t) = \tilde{\chi}_m(t)\underline{\underline{I}}$, $\tilde{\underline{\underline{\alpha}}}(t) = \tilde{\alpha}(t)\underline{\underline{I}}$, and $\tilde{\underline{\underline{\beta}}}(t) = \tilde{\beta}(t)\underline{\underline{I}}$ where $\underline{\underline{I}}$ is the identity dyadic. More important, this composite medium would have $\tilde{\alpha}(t) + \tilde{\beta}(t) = 0$ and would thus violate (2), if it could be made!

No one has been able to make a sample of the Tellegen medium. Elsewhere, it has been mathematically proved that even if such a

Manuscript received March 28, 1995; revised August 29, 1995.
A. Lakhtakia is with the Department of Engineering Science and Mechanics, Pennsylvania State University, University Park, PA 16802-1401 USA.
W. S. Weiglhofer is with the Department of Mathematics, University of Glasgow, Glasgow G12 8QW UK.
IEEE Log Number 9415593.

[1] Ari H. Sihvola, *Trans. Microwave Theory Tech.*, vol. 43, no. 9, Sept. 1995.

0018–9480/95$04.00 © 1995 IEEE

medium could be made, it would not be recognizable in modern electromagnetic theory because it would be indistinguishable from a simple dielectric-magnetic medium [5]. Similarly, a nonreciprocal bi-isotropic medium has been shown to be indistinguishable from a reciprocal bi-isotropic medium [7].

Apart from the mathematical and the existential evidences against the Tellegen medium, we also note that the physical conception of Tellegen contains a major flaw. When two bodies are brought in close proximity of each other, one can easily envision charge transfers taking place. In other words, a magnetic dipole and an electric dipole may not retain their individualities once they are glued together.

Charge transfers between the two would create new multipolar configurations, which could invalidate Tellegen's concept altogether.

III. Chromium Oxide

The second medium brought by Sihvola against (2) is a magnetoelectric medium: chromium oxide. In a detailed reinvestigation last year, it had been shown [6] that much of the work published in the 1960's was based on a wrong premise. Let us recapitulate briefly why that magnetoelectric literature does not stand the test of scrutiny.

The Lagrangian density $L(\underline{r},t)$ of an electromagnetic field is defined as

$$L(\underline{r},t) = \left[\tilde{\underline{B}}(\underline{r},t) \bullet \tilde{\underline{H}}(\underline{r},t) - \tilde{\underline{E}}(\underline{r},t) \bullet \tilde{\underline{D}}(\underline{r},t)\right] \tag{5}$$

into which (1a) and (1b) must be substituted. A causal medium satisfies the conditions

$$\tilde{\underline{\underline{\chi}}}_e(t) = \tilde{\underline{\underline{\chi}}}_m(t) = \tilde{\underline{\underline{\alpha}}}(t) = \tilde{\underline{\underline{\beta}}}(t) = 0, t \leq 0 \tag{6}$$

but no consequent simplification of (5) appears possible.

Suppose, however, a medium has purely instantaneous response, that is

$$\tilde{\underline{\underline{\chi}}}_e(t) = \tilde{\underline{\underline{\chi}}}_{eo}\delta(t), \tilde{\underline{\underline{\chi}}}_m(t) = \tilde{\underline{\underline{\chi}}}_{mo}(t)\delta(t), \tilde{\underline{\underline{\alpha}}}(t) = \tilde{\underline{\underline{\alpha}}}_0(t)\delta(t), \tilde{\underline{\underline{\beta}}}(t) = \tilde{\underline{\underline{\beta}}}_0(t)\delta(t) \tag{7}$$

where $\delta(t)$ is the Dirac delta function. Then $L(\underline{r},t)$ can be expressed only in terms of the instantaneous primitive fields, and the symmetry relations

$$\tilde{\underline{\underline{\chi}}}_{eo} = \tilde{\underline{\underline{\chi}}}_{eo}^T, \tilde{\underline{\underline{\chi}}}_{mo} = \tilde{\underline{\underline{\chi}}}_{mo}^T, \tilde{\underline{\underline{\alpha}}}_0 = -\tilde{\underline{\underline{\beta}}}_0^T \tag{8}$$

follow, with the superscript ^{T} denoting the transpose.

All the 1960's experimental work cited by Sihvola [3] on chromium oxide is based on (8). But (8) can hold only for media with purely instantaneous response—which are noncausal and do not exist.

Furthermore, a medium with instantaneous response is lossless. If one interprets (8) to hold at zero frequency, (8) cannot be extrapolated into the frequency domain. To illustrate this assertion, consider the following: (8) implies that the static permittivity dyadic of any material is symmetric, but the frequency-dependent permittivity dyadic of a lossless material is Hermitian. A Hermitian dyadic is not symmetric.

We conclude therefore that the mathematical integrity of (2) stands unchallenged and that the counter-arguments brought forth by Sihvola against (2) on physical grounds remain unsubstantiated. Several related clarifications have been made available elsewhere [10].

Acknowledgment

The authors would like to thank Ari H. Sihvola for sending us a preprint (dated February 20, 1995) of [3].

References

[1] E. J. Post, *Formal Structure of Electromagnetics.* Amsterdam: North-Holland, 1962.

[2] A. Lakhtakia and W. S. Weiglhofer, "Are linear, nonreciprocal, bi-isotropic media forbidden?" *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1715–1716, Sept. 1994.

[3] A. H. Sihvola, "Are linear nonreciprocal bi-isotropic media forbidden indeed?" *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2160–2162, Sept. 1995.

[4] A. Lakhtakia and W. S. Weiglhofer, "Constraint on linear, homogeneous constitutive relations," *Phys. Rev. E*, vol. 50, pp. 5017–5019, Dec. 1994.

[5] A. Lakhtakia, "The Tellegen medium is 'a Boojum, you see,' " *Int. J. Infrared Millim. Waves*, vol. 15, pp. 1625–1630, Oct. 1994.

[6] ——, "An investigative report on the constructive (*sic*) relations of linear magnetoelectric media," *Int. J. Infrared Millim. Waves*, vol. 15, pp. 1363–1372, Aug. 1994.

[7] W. S. Weiglhofer, "On a medium constraint arising directly from Maxwell's equations," *J. Phys. A: Math. Gen.*, vol. 27, pp. L871-L874, Dec. 1994.

[8] W. S. Weiglhofer and A. Lakhtakia, "Uniformity constraint on recently conceptualized linear uniaxial bianisotropic media," *Electron. Lett.*, vol. 30, pp. 1656–1657, Sept. 1994.

[9] A. Lakhtakia and W. S. Weiglhofer, "On a constraint on the electromagnetic constitutive relations of nonhomogeneous linear media," *IMA J. Appl. Math.*, vol. 54, pp. 301–306, 1995.

[10] W. S. Weglhofer and A. Lakhtakia, "A brief review of a new development for constitutive relations of linear bi-anisotropic media," *IEEE Antennas and Propagation Mag.*, vol. 37, pp. 32–35, June 1995. [Related correspondence appears in the Oct. 1995 issue.]

Author's Reply by Ari H. Sihvola

Call nonreciprocal bi-isotropic medium "Tellegen medium." As is clearly seen from the above [1], Lakhtakia and Weiglhofer absolutely reject the possible existence of Tellegen medium. My view—expressed in [2] and defended on the lines to follow—contradicts with this categorical result. I repeat the claim that conclusive measures to exterminate Tellegen media have not been demonstrated.

A. Mathematical Basis for the Equation Contradicting with Tellegen Medium

In this first section of [1], Lakhtakia and Weiglhofer note that if constitutive relations allowing for Tellegen medium are substituted into the sourceless Ampère–Maxwell curl equation, a redundancy emerges. The scalar material parameter responsible for isotropical nonreciprocity does not affect the propagating electromagnetic wave, which is a reason strong enough for Lakhtakia and Weiglhofer to conclude that this parameter (which I call "Tellegen parameter" in the following) does not exist.

To counter this, one needs only remember that there are several electromagnetic problems and structures where a certain parameter of the geometry or material of the problem does not affect the

Manuscript received August 22, 1995.
The author is with the Helsinki University of Technology, Electromagnetics Laboratory, Otakaari 5 A, FIN-02150 Espoo, Finland.
IEEE Log Number 9415592.

electromagnetic response, and we still do not claim that this parameter would not be real. Think of waves approaching planar structures. The existence of reflectionless coatings or Brewster angles does not mean that there was no interface.

But perhaps the gravest misunderstanding in this reasoning of [1] is that the redundancy only appears in the analysis of homogeneous media. If our world were filled with totally uniform Tellegen medium, with no boundaries nor heterogeneities, there would be little difference between this and another uniform universe built of reciprocal matter. A propagating light wave would continue on its eternal rectilinear path toward infinity with the same velocity as in a non-Tellegen medium.

But as soon as we have real-world inhomogeneities or interfaces between media that differ in their electromagnetic parameters from each other, then reflection, refraction, and diffraction phenomena arise. It is easy to see that if one repeats the above-mentioned redundancy reasoning for heterogeneous media, the spatial derivatives bring forth extra terms in which the nonreciprocal Tellegen parameter appears. The redundancy vanishes.

Recognizing the difficulty with the inhomogeneity, Lakhtakia and Weiglhofer refer casually to their article [3]. In that report, they take into consideration a sample of nonhomogeneous Tellegen medium, consider it as a union of piecewise homogeneous domains, and use their earlier redundancy result to eliminate the Tellegen parameter separately from each homogeneous subdomain. Well, what to do about the interfaces between the domains? The solution of [3] is to note that since the redundancy result applies on both sides of each interface, it is continuous across every interpiece boundary and, therefore, holds everywhere. Lakhtakia and Weiglhofer have wonderfully translated the properties of homogeneous media to arbitrary inhomogeneities.

B. Tellegen Medium

"No one has been able to make a sample of Tellegen medium." I do not know whether the claim is true, but I admit that I have not seen a real sample yet. However, this does not mean that it is not possible to make a sample. The proposition of [1] that Tellegen medium is indistinguishable from a simple dielectric-magnetic medium (meaning a medium with finite permittivity and permeability but with magnetoelectric parameters being zero) is clearly wrong.

To see why, consider a sample of Tellegen medium (Fig. 2 of [2]). There, pairs of permanent electric and magnetic dipoles are dispersed randomly in isotropic host medium. (The dipoles in each pair are parallel, in which case they produce a positive Tellegen parameter for the mixture, or antiparallel for a negative Tellegen parameter.) In response to incident electric field, the electric dipoles in the elements feel a torque, and an electric polarization is created. But at the same time, the magnetic dipoles also orient themselves because of the coupling to the electric dipoles, and a macroscopic magnetic polarization emerges. The fringing magnetic field due to this polarization can be measured in the surroundings of the sample. And here we have a method to distinguish Tellegen medium from a simple dielectric-magnetic medium. A dielectric-magnetic medium does not react to electric field excitation with a magnetic response.

The other suspicion of [1] against the Tellegen concept arises from the possible charge tranfers taking place as two dipoles are brought close together and the extra multipolar configurations thus created. It is true that a collection of multipoles responds to a field with complex spatial dispersion in a complicated way. But we do not need to use such a field to sense the Tellegen medium. Use an incident field that is static or at least uniform enough across the dimension of one dipole-pair element. Then only the dipole moment will react to this excitation. Higher-order multipoles do not feel an effect in a constant field and can be neglected.

C. Chromium Oxide

This material is antiferromagnetic, and it bears signifigance to the present discussion because it was in chromium oxide that the nonreciprocal magnetoelectric effect was first predicted and experimentally observed. An extremely active wave of research followed the discovery, and numerous publications, conferences, and textbooks were produced.

Now Lakhtakia and Weiglhofer claim [1] that this research has been based on a wrong premise, and the experimental works by Astrov, Folen, Rado, Stalder, and O'Dell "can only hold for media which are noncausal and do not exist." I leave it to the solid-state scientists to defend the rationality of one of their whole fields of study. On the reinvestigative report of the magnetoelectric studies ([6] in the bibliography of [1]) I have commented elsewhere in detail [4]. Be it here sufficient to note that with the strict interpretation of [1] of the absolute distinction between static and low-frequency problems, one misses real connections in electromagnetic solutions. For example, the attenuation of radio waves due to rain can be calculated to a very high accuracy using the polarizability of a raindrop in a static field. Quasi-static approach is also valid for magnetoelectric effects, where frequencies in the kilohertz region have been used. Translate this to wavelengths, and the spatial variation of field amplitude is definitely negligible across the sample under measurement. For a quantitative example, see [5] where magnetoelectric effects of cobalt-iodine boracite have been measured. In this report, Fig. 4(a) and (b) displays the static and dynamic (with the frequency of 160 Hz in the dynamical measurement) magnetoelectric coefficients as a function of temperature, and the two curves are almost indistinguishable.

Finally, I would like note that I have intentionally left out my explicit comments on many of the publications of Lakhtakia and Weiglhofer that are cited in [1]. In my opinion, little independent evidence against Tellegen media is provided by rewording the argumentation of [2] in [1] in tensor language ([4] in [1]), vector language ([5] and [7] in [1]), or dyadic language ([8] in [1]).

There would be interesting applications of Tellegen material in microwave technology [6]. With this reply, I try to maintain hope that one day we will be constructing devices that exploit the nonreciprocal properties in Tellegen media.

References

[1] A. Lakhtakia and W. S. Weiglhofer, Comment on "Are nonreciprocal bi-isotropic media forbidden indeed?" *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 12, pp. 2722–2723.

[2] A. H. Sihvola, "Are nonreciprocal bi-isotropic media forbidden indeed?" *IEEE Trans. Microwave Theory Tech.*, vol. 43, no. 9, pp. 2160–2162, Sept. 1995.

[3] A. Lakhtakia and W. S. Weiglhofer, "On a constraint on the electromagnetic constitutive relations of nonhomogeneous linear media," *IMA J. Appl. Math.*, vol. 54, pp. 301–306, 1995.

[4] A. Sihvola, "When doubting Tellegen material give her the benefit of the doubt," *Chiral Discussion Forum*, CHIRAL-L *at* listserv@DEARN.BITNET *alias* listserv@VM.GMD.DE, on Dec. 21, 1994.

[5] M. Clin, J.-P. Rivera and H. Schmid, "Low temperature magnetoelectric effects on $Co_3B_7O_{13}I$," *Ferroelectrics*, vol. 108, pp. 213–218.

[6] I. V. Lindell, A. H. Sihvola, S. A. Tretyakov and A. J. Viitanen, *Electromagnetic Waves in Chiral and Bi-Isotropic Media.* Norwood, MA: Artech, 1994.